

## **Bayesian operational modal analysis of offshore rock lighthouses for SHM**

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### **Abstract**

During 2016 and 2017 a program of field vibration measurements was made on a set of Victorian era granite lighthouse towers around the British Isles. The field tests were designed for structural identification to enable condition assessment and identification of extreme wave loads through long term monitoring. The primary test method was forced vibration, and ambient vibration measurements was used as a backup. The best operational modal analysis (OMA) results were obtained using Bayesian OMA, which provide a clear picture of the directionality of the mode shapes which appeared at very close frequencies due to the symmetry of the towers. The paper describes measurements and sample analysis illustrating difficulties and achievements.

### **1. Introduction**

Despite widespread use of virtual navigational aids such as GPS, visual navigational aids such as lighthouses are still needed to protect mariners and preserve trade. These are the same motives for lighthouse construction for the last two or three millennia, but it is apparently only in the 17th century that lighthouses were first constructed on the dangerous offshore rock outcrops causing multiple shipwrecks. Apparently the first example was the lighthouse constructed on Eddystone Reef in 1698, in the south west approach to Plymouth by Henry Winstanley. He had the right to collect all dues from ships passing the light for the next 5 years, then half dues for 50 years before all dues were to Trinity House. The same business model is still operated by Trinity House and the two other General Lighthouse Authorities operating around the British Isles i.e. The Northern Lighthouse Board and Commissioner of Irish Lights. The present-day Eddystone Lighthouse designed by James Douglass was completed in 1882 (1). Douglass also designed the Les Hanois (1862) lighthouse off Guernsey and Longships Lighthouse (1875), close to Land's End, while Wolf Rock (1869) and Bishop Rock (1858 and 1887) lighthouses further southwest were designed by James Walker.

The five lighthouses (Eddystone, Les Hanois, Longships, Wolf Rock and Bishop Rock) are all concave elliptic frustums constructed from dovetailed granite blocks and were all retrofitted with steel frame helidecks between 1973 and 1981.

## **2. STORMLAMP project definition**

Project STORMLAMP: *STructural behavior Of Rock Mounted Lighthouses At the Mercy of impulsive waves*, funded by EPSRC, was initiated in 2016 with the aims to

- Identify experimentally modal parameters of a set of at least six rock lighthouses
- Monitor dynamic performance of at least one lighthouse over an extended period
- Develop structural models based on construction data and dynamic testing
- Investigate worst case hydrodynamic loading due to breaking waves and
- Formulate guidance for structural condition assessment and management

### ***2.1 Helideck-equipped lighthouses: Logistical challenges.***

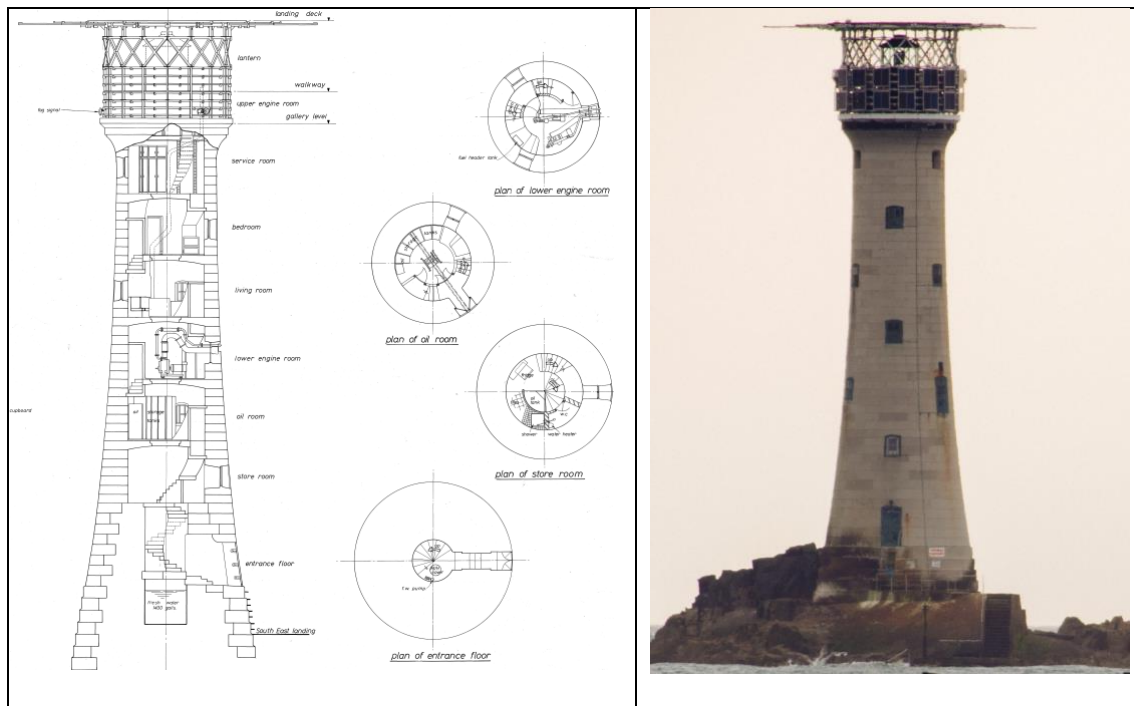
Five of the lighthouses studied for STORMLAMP are the five previously mentioned that are located in the English Channel and Atlantic approaches and which are the subject of this paper. Lighthouses at Fastnet Rock (Ireland), Dubh Artach (Scotland) and Skerryvore (Scotland) are being studied, but these lack the retrofitted helidecks and as such are technically less challenging, from the point of view of operational modal analysis (OMA). Logistical challenges of the helideck-equipped lighthouses are also significant due to operational limits of the aircraft used to access them. Total weight of passengers and freight is limited and they operate according to visual flight rules which prevent them flying in foggy weather which often envelopes lighthouses. Experimental field campaigns require a return trip in one day or an overnight stay and are synchronised with visits of GLA maintenance teams that take priority so flights can be re-timed or cancelled at short notice with consequences on the experimental work.

### ***2.2 Modal test planning: Signal to noise ratio challenges***

The only prior information to inform test planning was response monitoring of Eddystone Lighthouse (2) using geophones, which had indicated a fundamental frequency around 4.4 Hz. Electrodynamic shakers such as APS 113 and APS 400 operate optimally at this frequency but Douglass' paper (1) gives total mass of granite as  $4.8 \times 10^6$  kg which represents a structure usually regarded as being too massive to 'get going' with a shaker. The modal mass with mode shapes unity scaled at the top of the tower (where the shaker would operate) could be much less and a reasonable signal to noise ratio might be obtained using the larger shaker. However only the smaller APS 113 could be safely handled on site so it was highly likely that ambient vibration testing would be the best option. Hence the first modal test in the series was designed to use both ambient and modal test methodologies, to check how well either of these methods worked, and to adapt future tests according to what worked best.

### 3. Les Hanois Lighthouse modal test

A three-person test crew left Guernsey Airport at 1PM (2 June 2016) and returned by 6PM. Allowing for a 5 minute helicopter flight, unloading, setting out equipment and packing up this left 2-3 hours of for modal testing on a structure unlike any the test team had experienced. The ten levels for measurement are shown in Figure 1 and include two levels resulting from the helideck retrofit. Data acquisition equipment comprising a 24 channel 24-bit Data Physics spectrum analyser running at 204.8 Hz and accelerometer power supplies was set up in level 8 (battery room). At level 9 (lantern room, gallery level) the shaker was set up on the external gallery, which also provides access inside the lighthouse from the helideck above, with a pair of accelerometers arranged as references in orthogonal directions inside the lantern room.



**Figure 1: Les Hanois Lighthouse layout (Trinity House) and view from land (1 km away).**

The remaining 10 accelerometers were arranged as orthogonal pairs at level 1 (the lowest, at the emergency exit to the reef), level 2 (store, that is also the bathroom), level 5 (living room), level 6 (bedroom) and level 10 (helideck). Low-noise accelerometers not relying on cables, GPS timing or wireless communication were not at the time available so cabled Honeywell QA-750 quartz-flex accelerometers were arranged, with careful cable management along the interior wall and down the spiral staircases. Sensors were arranged at the same compass bearing with respect to the lighthouse vertical axis, making use of reference features such as windows, and aligning them tangential and perpendicular to the lighthouse inner wall with the help of a wooden jig so that X-axis pointed in northeast direction and Y-axis in northwest direction.

This arrangement was used for the first set of measurements, 'swipe 1'. For the second arrangement 'swipe 2', accelerometers at levels 1 and 2 were moved to levels 3 (oil room) and 4 (lower engine room), and those at levels 5 and 6 to levels 7 (service room) and 8

(upper engine room/battery room). The aim of using two swipes with orthogonal accelerometers was that the two reference pairs at levels 9 and 10 would enable assembling or ‘gluing’ of mode shape pieces in the modal analysis process.

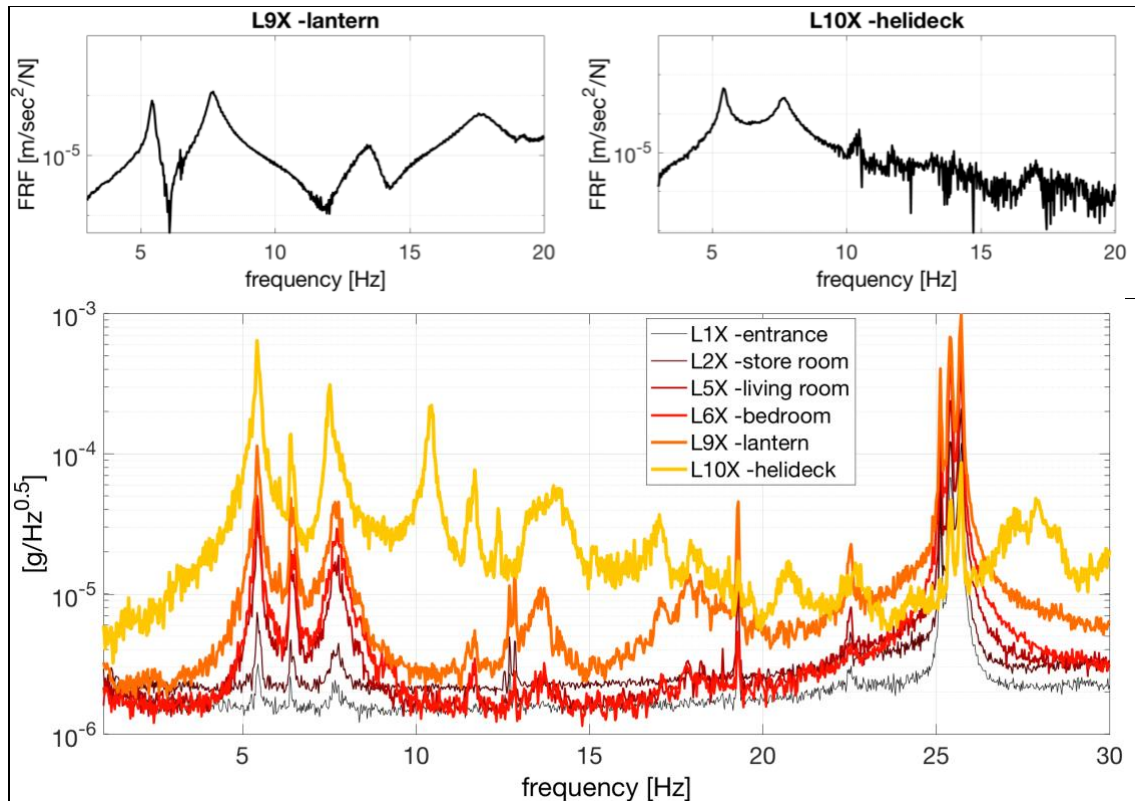
Measurements are summarised in Table 2. These exclude system checks and unsuccessful measurements and account for 70.5 minutes of good quality measurements. In each swipe a series of individual measurements (runs) were made, varying shaker conditions or recording ambient response.

**Table 2: Les Hanois Lighthouse measurement sequence**

Run	Swipe levels		Shaker direction	Excitation	Duration/ s
5	1	1,2,5,6,9,10	-	ambient	940
6	1	1,2,5,6,9,10	y	Swept sine 3-20 Hz	690
7	1	1,2,5,6,9,10	x	Swept sine 3-20 Hz	640
13	2	3,4,7,8,9,10	y	ambient	340
14	2	3,4,7,8,9,10	x	Swept sine 3-20 Hz	720
17	2	3,4,7,8,9,10	y	Swept sine 3-20 Hz	600
19	2	9,10	y	Swept sine 5-6 Hz	300

### 3.1 Comparing ambient and forced vibration response

Data from forced and ambient vibration measurements (run7 and run5 respectively in swipe 1) are shown in Figure 2. The figure characterises issues in the two approaches.



**Figure 2: Les Hanois FRF for forced vibration testing (upper plot) and auto-spectra for ambient response (lower plot).**

Les Hanois is, with Longships, one of the two smaller lighthouses, 36 m tall from base to helideck. It was tested in relatively calm conditions, in other words signal to noise ratios should be good for system identification using forced vibration data. Achieving a good signal to noise ratio requires an appropriate forcing function, which is a choice not required in ambient vibration measurements. Data Physics Signalcalc software was used, initially using a broadband random excitation, as is conventionally used with shakers, but this proved to be a poor choice for Les Hanois, with poor signal to noise ratio. Swept sine (sweeping up then down a limited band) proved to be more viable, with appropriate choice of sweep band and window length allowing sufficient resonant build up around modes. Repeating the sweep to average out noise meant trade-offs with resolution and signal to noise ratio within tight time constraints.

The frequency response functions (FRFs) in Figure 2 are for swept sine excitation in the range 3-20 Hz with 40 s window and apparently show two clear vibration modes in the 'X' direction in the range 5-10 Hz. System identification was tried using both circle fitting and global rational fraction polynomial (GRFP). Circle fit is a highly visual method where a clear fit to a single modal circle in a Nyquist plot can be very convincing, but in fact misleading, whereas GRFP requires judgement of number of modes and produces different results depending on the number of modes fitted. For Les Hanois using a narrow sweep range 5-6 Hz did not provide much improvement proving the choice of shaker signal to be adequate in this case. For larger structures during windier conditions at other structures tested (e.g. the much more massive Bishop Rock lighthouse) the signal to noise ratio was significantly reduced. For Bishop Rock the shaker used at Les Hanois was damaged (possibly during transport) and the replacement had a lower power output. Running a shaker was always a problem due to logistics as well as failures and time spent chasing the problem, but the benefit was direct measurement of modal mass, a vital parameter for inverse identification of wave loading from response data. Modal mass cannot be directly estimated using OMA.

Modal test results from forced vibration testing are reported in full elsewhere, so this paper focuses on using ambient vibration data in OMA. OMA has the advantage of not needing to control or measure the excitation force so the trade-offs are essentially simpler: frequency resolution and number of averages for a fixed time, the sum of which (in conventional OMA) leads to bias and variance errors in estimates of modal parameters (MPs). Bayesian methods simplify the process, and guidance on test planning is now available to achieve specified precision in MP estimation (3).

The ambient response (Figure 2), shown as a power spectral density (PSD) function appears to be rich in information. While ambient response does not depend on shaker bandwidth or position (to drive all modes) it might be coloured by excitation sources such as internal generator. The PSD shows an extra mode in the 5-10 Hz range, a probable mode just over 10 Hz and a further set of probable modes, one of which (~13 Hz) is consistent with the FRFs. Around 25 Hz is a set of three apparent modes that are believed to be mechanical response due to the generators, since while on station at Les Hanois the audible tone of the generators (that powers the lighthouse when manned) was hunting between three different speed bands around 1500 revolutions per minute. The sharpness of the peaks is also not consistent with the other modes.

### 3.2 Bayesian OMA

Bayesian operational modal analysis (BAYOMA) (4) yields the probability density function of modal properties (MP) using the FFT of ambient vibration data in a selected frequency band around the subject modes. In a Bayesian context, given the data, the modal properties are approximately Gaussian. The most probable value (MPV) of MP minimises the negative of the log-likelihood function (NLLF), and their covariance matrix is the inverse of the Hessian of the NLLF. Efficient algorithms have been developed for calculating these quantities. In BAYOMA, the modes are assumed to be classically damped, and the noise and modal force PSDs are assumed to be independent and to have constant PSDs within the selected frequency band. BAYOMA identifies the real mode shapes (in contrast to operational deflection shapes), which need not be orthogonal as they are confined to the measured degrees of freedom only.

Once the mode shapes in each setup are identified (in terms of their MPV), they are glued together using a global least square method (5) that minimises (under norm constraint of glued mode shape) a quadratic measure-of-fit function accounting for the discrepancies in all setups. For challenging cases of poor s/n ratio in some setups leading to erroneous results from least square methods, a multi-setup Bayesian algorithm may be adopted as it is found to give more robust results, although currently it can only be done efficiently when the modes are well-separated (6), which is not the case for lighthouses. This because lighthouses are almost (but not quite) axisymmetric and in the limiting case of perfect axis-symmetry could exhibit omnidirectional mode shapes.

Figure 3 shows six identified mode shapes for Les Hanois, including the mode not visible in the forced vibration FRF. The elevation view projects the modes in the best fit plane of vibration which is identified in the plan view. In the elevation view the mode shapes are normalised to (positive) unity in the helideck modal ordinate. A feature which recurs in all the lighthouses is that there are at least four modes having a zero-node cantilever character in the masonry tower. This is partly due to the symmetry resulting in mode pairs with approximately orthogonal direction when seen in plan, and partly due to the helideck which behaves to some extent as a lumped mass on a spring and which splits what might be a single mode in a simple tower into two modes. These modes have alternate phase ( $0^\circ$  or  $180^\circ$ ), a behaviour rather similar to that of a structure with a tuned mass damper.

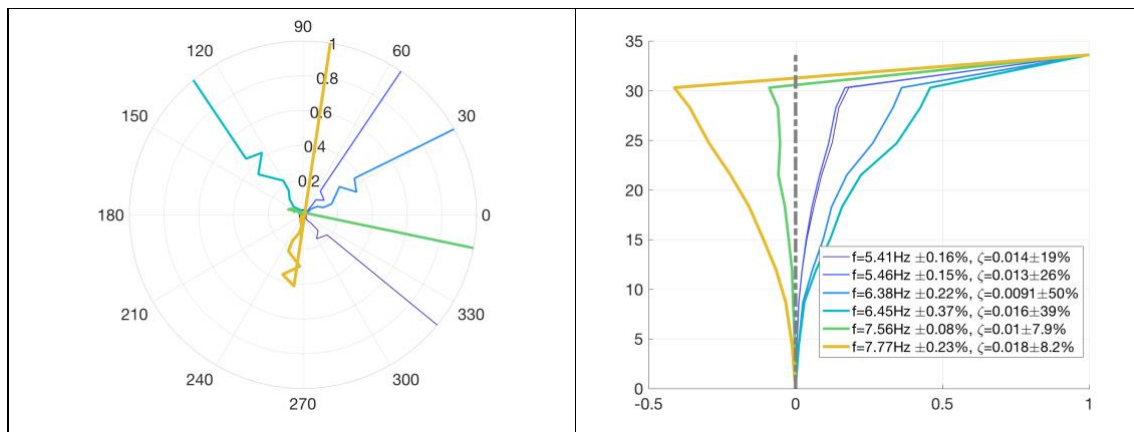


Figure 3: Les Hanois mode shapes. Left: in plan. Right: in elevation



For Les Hanois, there are two additional modes with the  $0^\circ$  phase of the helideck ordinate, which do not make sense structurally; while many of the peaks in the ambient response PSD appear only in the helideck this pair has clear response in the masonry tower. Figure 3 also indicates the uncertainty in the MP estimates in terms of coefficient of variation (standard deviation/mean). As usual, frequency estimates have low uncertainty while damping (given as ratio, not percentage) is significantly larger, although in the case of lighthouses damping ratio is not a parameter having major influence on response.

#### 4. Wolf Rock lighthouse

Wolf Rock Lighthouse lies 15 km southwest of Land's End, and is built on a rock outcrop rising sharply 37 m from the sea bed. It is exposed to extreme wave loading mainly from the Atlantic Ocean in the southwest direction. In addition, anecdotal evidence from lighthouse keepers suggests that it experiences lively dynamic response (rocking) during storms. The modal test procedure for Wolf Rock was very similar to that at Les Hanois, with similar high-pressure timescales, and Figure 4 shows the ambient response PSD for one measurement. This PSD is shown for X-direction, which was chosen for logistical convenience when planning the measurements (based on Trinity House drawings), there being no prior knowledge of principal directions of major and minor stiffness.

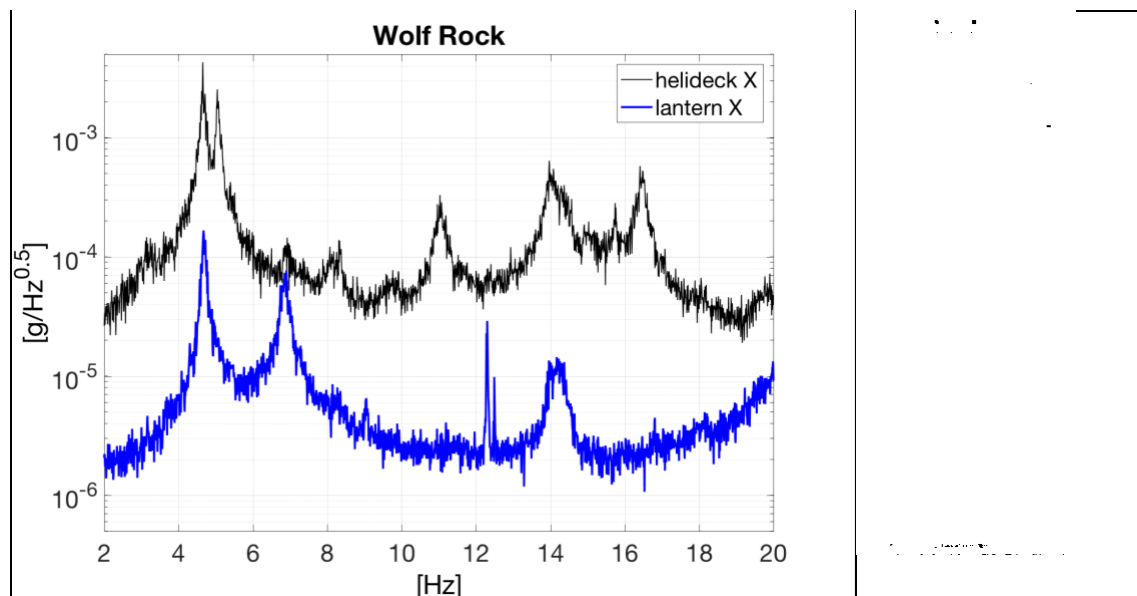


Figure 4: Wolf Rock Lighthouse (Trinity House) and ambient response PSD.

As with Les Hanois, the helideck response is much stronger than the masonry structure *except for the second mode* and there are at least two X-modes in the low frequency range (0-10 Hz) plus an extra peak of helideck-only response. Mechanical problems with the shaker meant that 15 minutes of ambient response were obtained for swipe 1, but only 64 s of data were available for swipe 2. The shorter swipe 2 duration resulted in greater uncertainty and the mode shapes (Figure 5) are jagged in elevation and non-orthogonal (as pairs) in plan. Even so, the essential features of  $0^\circ$  or  $180^\circ$  phase angles for sub-0 Hz modes are visible. Forced vibration testing has the advantage of better mode shape identification as well as modal mass estimation, and one result is that for unity scaling of mode shapes at the top of the masonry tower, the modal mass of the first mode ( $\sim 4.6$  Hz)

is very much larger than that for the second mode ( $\sim 6.8$  Hz). The 14 Hz mode does not have a node in the tower; the first mode exhibiting this character is at 21 Hz.

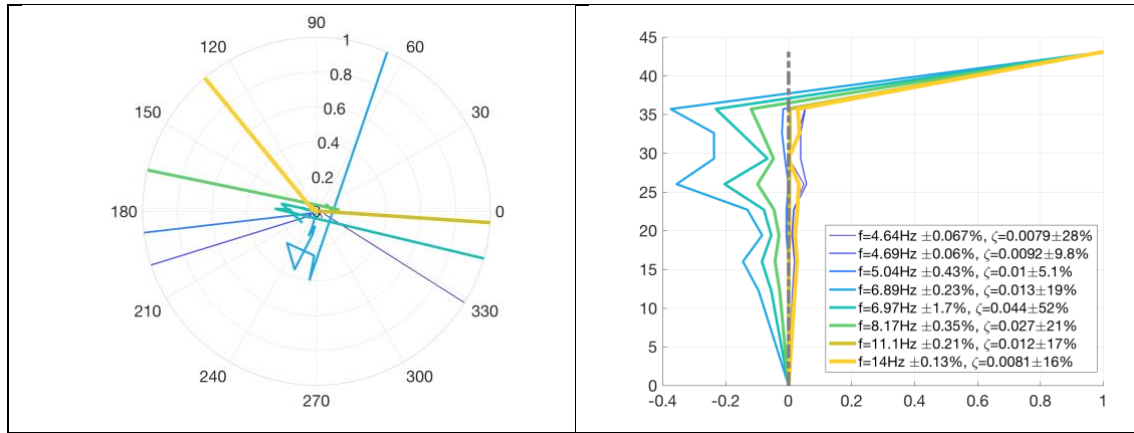


Figure 5: Wolf Rock mode shapes. Left: in plan. Right: in elevation

## 5. Eddystone

Eddystone Lighthouse lies 21 km southwest of Plymouth, and is the fourth structure on the site. It is also a giant, with a total of 11 levels and height of 49 m. The modal test on Eddystone was the most successful among all the lighthouses because lessons from pervious tests had been learnt, because the lighthouse had been visited by STORMLAMP crew previously, and because the test team had the luxury of being able to continue measurements overnight and the morning of the second day. The test procedure had evolved after Wolf Rock to laying out accelerometers one per level in X-direction for one swipe then rotating all accelerometers (with the shaker) to the Y-direction. The smooth PSD of X-direction response (Figure 6) is the result of averaging signals from an overnight recording and shows at least six vibration modes in the range 0-10 Hz.

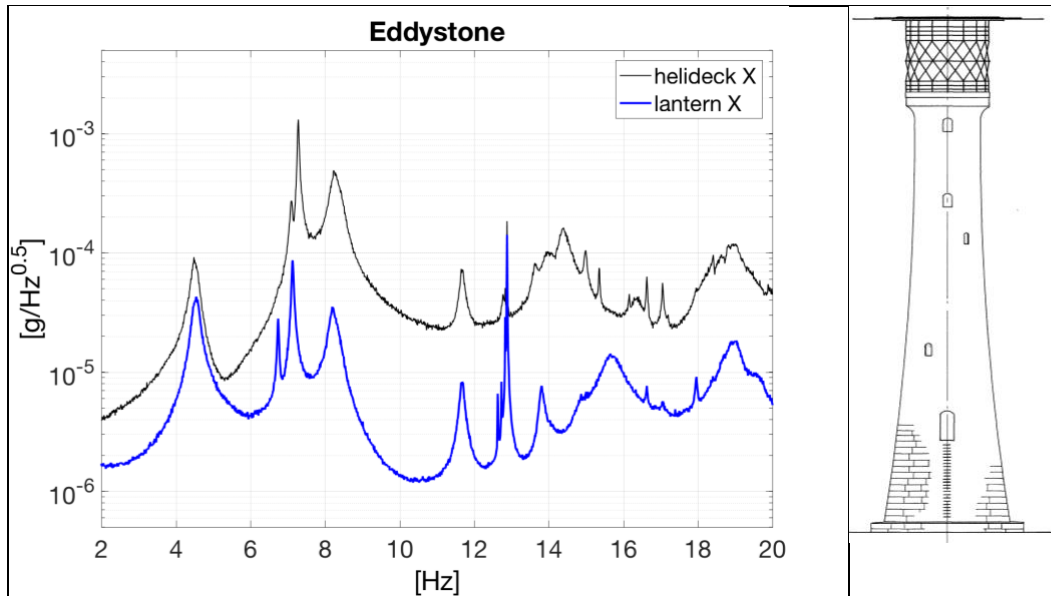
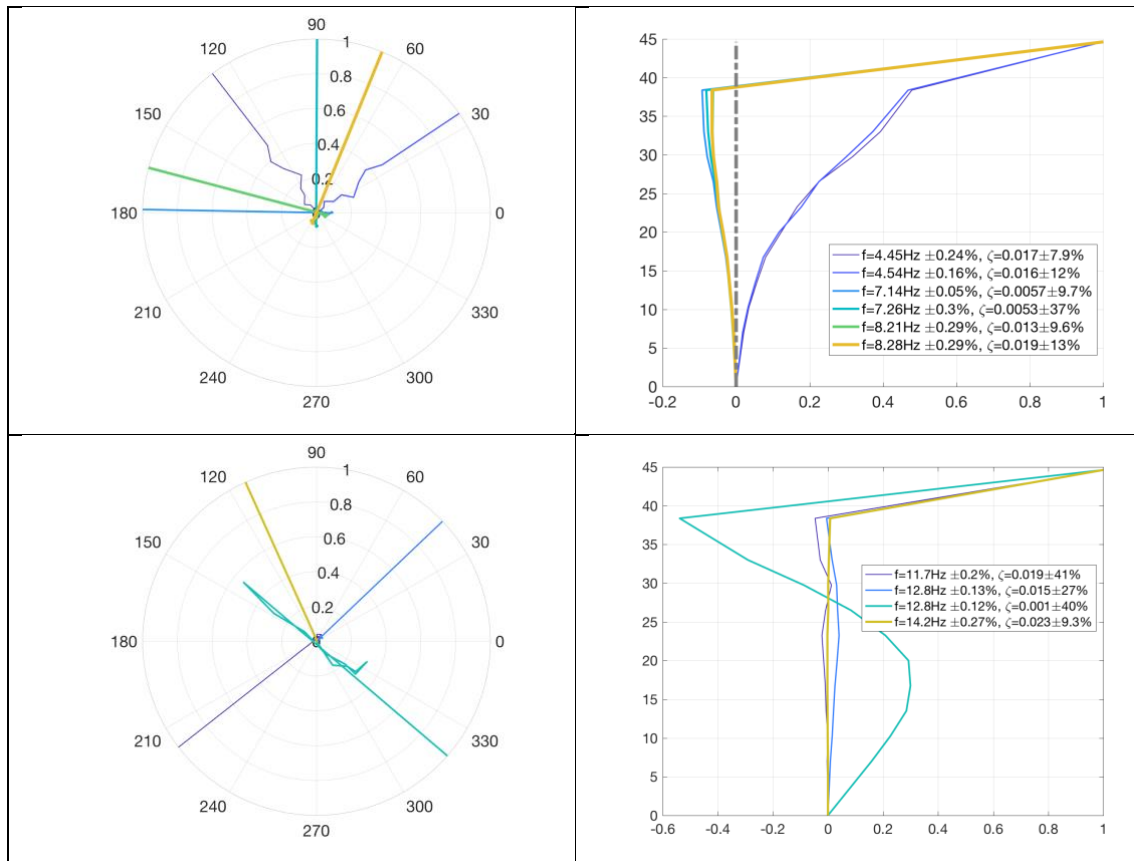


Figure 6: Eddystone Lighthouse (Trinity House) and ambient response PSD.



The forced vibration test only finds the modes around ~4.5 Hz and ~8.2 Hz, yet the mode shapes from BAYOMA (Figure 7) are very clear and show that the pair around ~7.2 Hz have the same shape (180° phase with the helideck) as do the modes around ~8.2 Hz, although their damping ratio is suspiciously low. Different from Wolf Rock, the lower frequency mode pair has the smaller ratio between tower and helideck modal ordinates compared to the upper frequency.

The set of four higher frequency modes in the lower plots show the second order (single node) mode shape at 14.2 Hz (identified in the forced vibration testing at 15.7 Hz) and a mode at 12.8 Hz with impossibly low damping ratio.



**Figure 7: Eddystone mode shapes: Left: in plan. Right: in elevation**

As for Les Hanois, the mode shape alignment for the first six modes in the upper plots of Figure 7 show orthogonality in plan. This alignment is very difficult to identify with forced vibration testing because almost any alignment of (orthogonal) X and Y axes will produce apparently sensible results in terms of clear FRFs. It is possible, but somewhat challenging, to find alignment for the forced vibration test but OMA offers a clear advantage in this sense, and is also better choice for estimating modal frequencies for a pair of very close modes.

## 6. Helideck vibration modes

As the helideck vibration modes figured significantly in the behaviour of all the helideck-equipped lighthouses, time was available on station at Eddystone to map out horizontal mode shapes at the landing deck (helipad) level based on a short ambient recording using four QA 750 accelerometers arranged on the helideck. The data were analysed using eigensystem realisation algorithm (based on cross-covariance functions from the time series), a method that does not provide directly for uncertainty quantification, and the result is shown in Figure 8. The six modes (in pairs) below 10 Hz are identified, all being essentially translational and with obvious orthogonality for the  $\sim 4.5$  Hz and  $\sim 8.2$  Hz mode pairs, while the two highest frequency modes are torsional. This makes sense from the structural point of view due to the framing system for the helideck, and also suggests why some modes do not appear in the masonry tower response.

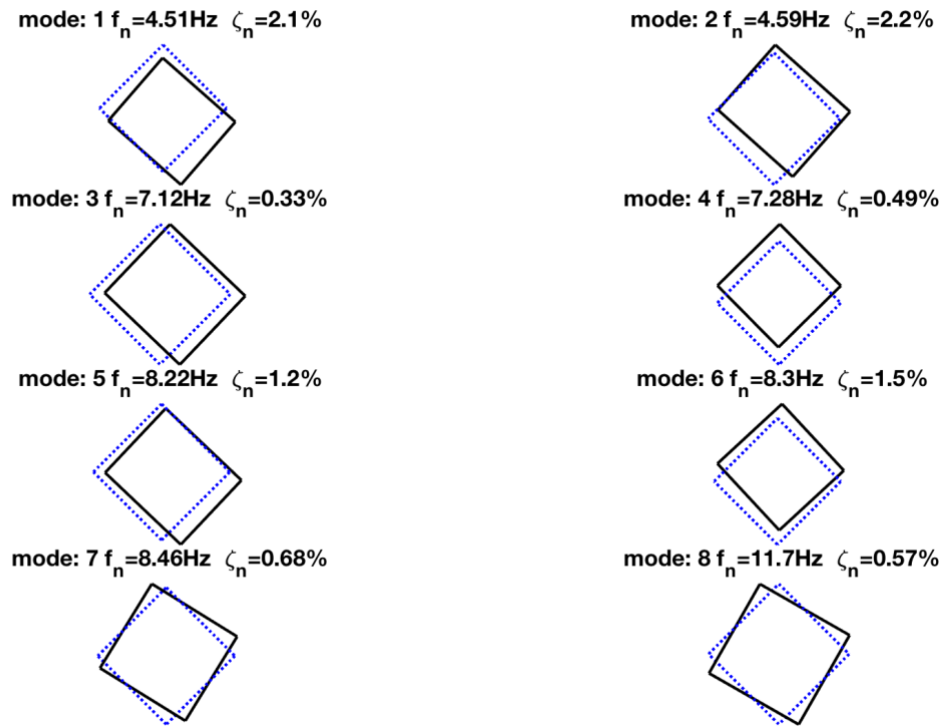
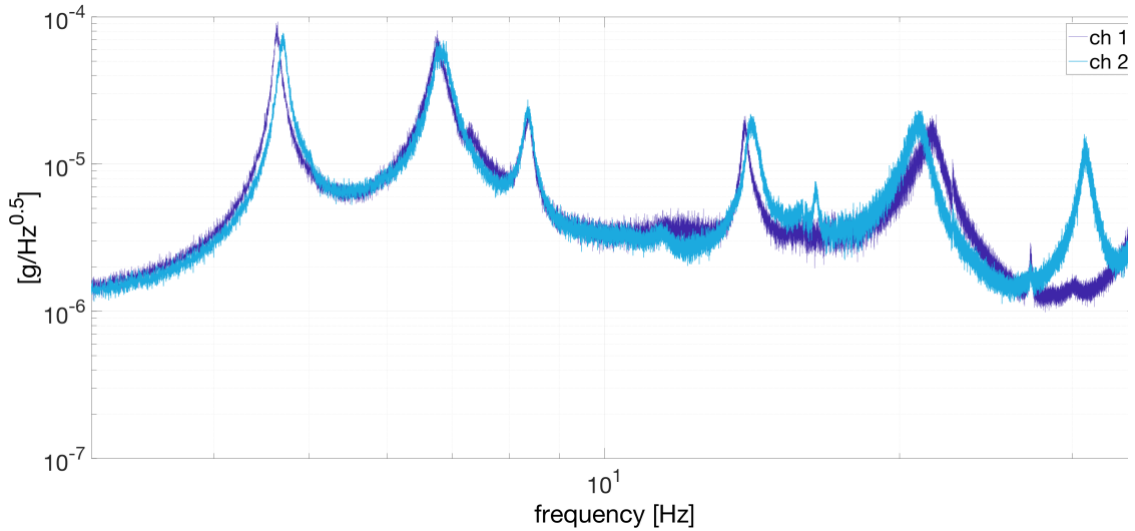


Figure 8: Eddystone helideck mode shapes.

## 6. Long term monitoring

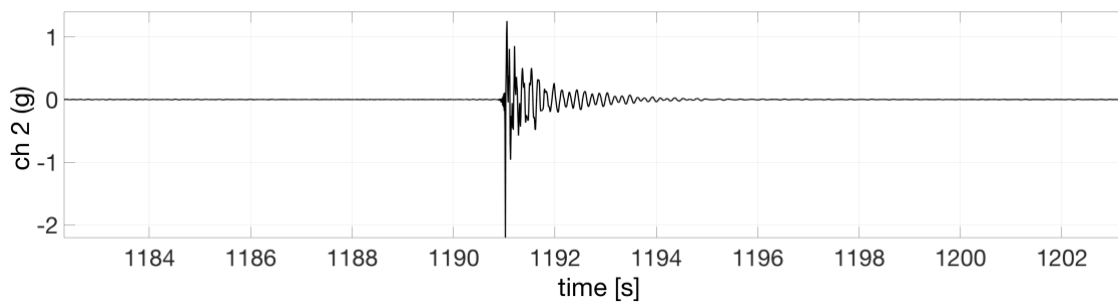
Based on the experience of modal testing a set of rock-mounted lighthouses, Wolf Rock was chosen as the ideal candidate for long term monitoring. Wolf Rock is one of the smaller structures (so should respond more strongly to loading), it apparently rocks on its foundations during extreme loads, and its location and bathymetry leaves it exposed to possibly the strongest breaking wave loads of all the lighthouses. Hence a single triaxial servo-accelerometer was left on station, fixed to the floor of the battery room with a National Instruments (CompactRIO) data acquisition system and with data streamed by 3G/4G to a land-based server. The system was installed in September 2017, but the data

connection was lost until repaired in a visit in February 2018, when the winter data were retrieved. The period featured several major storms including Storms Aileen, Ophelia (formerly a hurricane), Brian and Eleanor. Figure 9 shows response for two horizontal (orthogonal) directions during one day of significant (but not extreme) wave loading (20th September). There is clear structural response in the  $\sim 8$  Hz mode although the majority of response is in both  $\sim 4.6$  Hz and  $\sim 6.8$  Hz modes. The small differences in the mode frequencies for each direction reflect the symmetry, although there is no certainty that the sensors are aligned in the principal directions.



**Figure 9: PSD of ambient response during one day of monitoring or response to wave impact.**

The strongest response was observed during Storm Brian (21st November). The free decay is strongly non-linear and would not be amenable to OMA that assumes linear behaviour. The response to this and other storms will be 'decoded' to interpret wave loads based on the results of both forced and ambient testing.



**Figure 10: Extreme wave load response during Storm Brian.**

## 8. Conclusions

Modal analysis of offshore rock lighthouses is challenging in several technical respects as well as the logistical ones. The usual short time on station works against the usual requirement for long data length for most accurate modal parameter estimation. The BAYOMA MP identification has been extended (3) by research (of which this experimental study is a part) that informs the data length for the required precision. Even

though the relatively high natural frequencies benefit MP identification from OMA, there are challenges due to the axi-symmetry, which lead to imprecise and difficult to identify plan alignment of mode shapes and to very close mode frequencies. In fact realistically, only OMA can identify the alignment directly and this has been done successfully in this study.

BAYOMA provides extra confidence in MP estimation via the uncertainty quantification, and this exercise demonstrates the need to quantifying uncertainty in the alignment, which is the aim of further research.

Bayesian OMA (as for other techniques such as stochastic subspace identification and eigensystem realisation algorithm) suggests modes that forced vibration testing cannot find, but the challenge is to find if those modes are significant in terms of operational response in the case of extreme wave loading. The modal test data obtained for Wolf Rock is now being used to interpret response data from a pair of accelerometers permanently installed in the masonry tower

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## **References**

- [1] W.T. Douglass, S. Webb, G.W. Owen, J.B. Redman, M. Beazeley, L.F. V Harcourt, et al., Discussion. The new Eddystone Lighthouse., Minutes Proc. Inst. Civ. Eng. 75 (1884) 37–56.
- [2] A. Raby, G.N. Bullock, D. Banfi, Y. Rafiq, F. Cali, Wave loading on rock lighthouses, Proc. Inst. Civ. Eng. - Marit. Eng. 169 (2016) 15–28.
- [3] S.K. Au, J.M.W. Brownjohn, J.E. Mottershead, Quantifying and managing uncertainty in operational modal analysis, Mech. Syst. Signal Process. 102 (2018) 139–157.
- [4] S.K. Au, Operational Modal Analysis: Modeling, Bayesian Inference, Uncertainty Laws, Springer, 2017.
- [5] S.K. Au, Assembling mode shapes by least squares, Mech. Syst. Signal Process. 25 (2011) 163–179.
- [6] S.K. Au, F.L. Zhang, Fast Bayesian Ambient Modal Identification Incorporating Multiple Setups, J. Eng. Mech. 138 (2012) 800–815.